CAT 2023

**Question 1**

1. Distinguish between numerical and analytical methods in the solution of mathematical problems.

(3 Marks)

1. Describe the bisection method and explain how it differs from the Newton-Raphson method.

(3 Marks)

1. Given an initial guess *x=3*, find an approximate value of the root of the function using 3 iterations. Provide a sample R code that you would use to solve this problem, stopping after 40 iterations.

(6 Marks)

1. Using the inverse-transform approach to explain (mathematically) how you would generate random numbers from the exponential distribution, and further provide an R code that will be used to generate random numbers from this distribution.

(8 Marks)

**Question 2**

Starting with the Newton-Raphson formula

shown that the order of convergence of the Newton-Raphson method is

as with provided

(20 Marks)

**Question 3**

1. Consider the following data:

|  |  |  |  |
| --- | --- | --- | --- |
| x | 1 | 2 | 4 |
| y | 1 | 3 | 1 |

Use quadratic spline interpolation to find the approximate value of y at x=3.

Hint:

(10 Marks)

1. Consider the following data

Use polynomial interpolation to determine the value of the function at x=2.7.

Hint:

(10 Marks)

**Question 4**

Consider the general linear model

whereis an vector of response values, is an design matrix corresponding to the explanatory variables , is the vector of parameters, and is the variance.

Using a maximum likelihood approach, clearly showing the likelihood function, the log-likelihood function, and the score-vector, derive the maximum likelihood estimators of and .

(20 Marks)

**Question 5**

1. Show that the probability density function of the Poisson distribution, belongs to the exponential dispersion family

(3 Marks)

1. Also show that the mean and the variance of the Poisson distribution are equal to and , respectively.

(3 Marks)

1. Consider the generalized linear model

whereis an vector of response values belonging to the Poisson distribution, is an design matrix corresponding to the explanatory variables , is the vector of parameters.

Derive an expression for the estimating equation and Hessian that would be used to estimate the vector of parameters

(14 Marks)